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MEMORANDUM
RM-3014-PR
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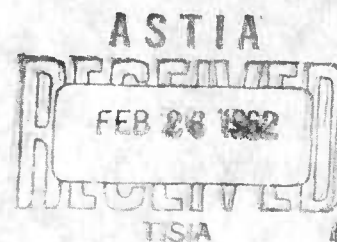
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WHEN TO STOP SAMPLING INITIATE PRODUCT IMPROVEMENT

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PREPARED FOR:

UNITED STATES AIR FORCE PROJECT RAND

The **RAND** *Corporation*
SANTA MONICA • CALIFORNIA

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PREFACE

This Memorandum is one part of a continuing study concerning the relation between information collection and system effectiveness. The relation studied here is that between information collection and the product improvement activity. This topic will be treated more completely in a future Research Memorandum. This Memorandum is being distributed now to a limited audience because of its possible relevance to current Air Force interest and actions in the area of product improvement.

SUMMARY

This Memorandum considers the following problem. Funds of a certain amount are made available for the improvement of a particular weapon system. The weapon system is composed of several subsystems which fail independently. Each of these subsystems is an investment prospect for product improvement. If information regarding the attractiveness of each of these prospects were free, cost and return estimates would be obtained for each of the subsystems and the funds would be allocated on the basis of these estimates. However, such information is frequently costly. When this is the case, it is usually uneconomical to purchase cost and benefit estimates for each subsystem. The funds expended on excessive information gathering could have been more profitably invested in actual product improvement. A rule is required which will indicate when the decision maker should stop gathering information and initiate product improvement. The purpose of this Memorandum is to provide such a rule.

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I. INTRODUCTION*

Product improvement is a term used by the Air Force to describe the process by which resources are expended to enhance weapon system performance. It is assumed here that performance is improved whenever the cost of producing a unit of weapon system readiness is reduced. This cost includes the downtime costs associated with the various repair actions as well as the dollar cost of these actions. These improvements are assumed to occur within the given management system. An in-commission or ready weapon system is assumed to be capable of performing its mission.

The following problem is considered. Funds of amount K are made available for the improvement of a particular weapon system. The weapon system is composed of n subsystems, which fail independently. Each of these subsystems is an investment prospect. If information regarding the attractiveness of each of these prospects were free, cost and return estimates would be obtained for each of the n subsystems and the K funds would be allocated on the basis of these estimates.** However, such information is frequently costly. When this is the case, it is usually uneconomical to purchase cost and benefit estimates for each subsystem. The funds expended on excessive information gathering could have been more profitably invested in actual product improvement. A rule is needed which will tell the decision-maker when to stop gathering information and initiate product improvement. The purpose of this Memorandum is to provide such a rule.

* I wish to thank H. Markowitz and R. L. Van Horn for several very helpful suggestions.

** For a solution to this kind of allocation problem see Ref. 1 and Ref. 2.

II. A STOPPING RULE WHEN THERE IS NO PRIOR INFORMATION OF COSTS AND RETURNS

Resources of amount K are available for improving a particular weapon system. The weapon system is composed of n subsystems. A maximum of k_1 of the K resources can be invested in the i -th subsystem at a rate of return, ρ_i , $i = 1, 2, \dots, n$. Any amount less than k_1 can also be invested at the same rate of return. The variables, k_1 and ρ_i , are assumed to be random with known distributions. The random variable, k_1 , is distributed by $\Phi_1(k_1)$ and the random variable, ρ_i , is independent of k_1 and distributed by $\Phi_2(\rho_i)$, where Φ_1 and Φ_2 are known. At a price of I the decision maker can purchase information on the exact values of k_1 and ρ_i for any i . From the point of view of Φ_1 and Φ_2 , I can be regarded as the cost of drawing a sample of size one from each distribution. In terms of the Air Force product improvement program, I can be interpreted as the cost of obtaining engineering estimates of the improvement that can be anticipated for the i -th subsystem and the associated cost. Such estimates are a necessary prelude to any improvement project. Since it is assumed that no prior information is available on ρ and k_1 for any subsystem, the decision maker is indifferent to whether information is purchased first for subsystem i and then possibly, if the stopping rule permits, for subsystem j or first for subsystem j and then possibly for subsystem i . The next section considers the case where there is some prior information on returns and, therefore, a prior ranking of candidates.

Suppose that the K funds could be invested at a rate of return, ρ_0 , in some project, P_0 , other than the improvement of the weapon system under consideration. Without incurring any informational cost, a return of $\rho_0 K$ can be had with certainty. The alternative is to purchase information on

ρ_1 and k_1 for the i -th subsystem. The cost of this information is I . It is assumed that after each observation, say the m -th, on ρ_1 and k_1 , the decision maker can decide whether to purchase another observation for I dollars or to stop sampling and invest the $K - mI$ funds in the known investment prospects.* The following rule, exemplified for the first stage decision, will be used to decide between these two alternatives:

If the expected return from the investment opportunities available after spending I exceeds $\rho_0 K$, purchase information on ρ_1 and k_1 .

If the expected return from the investment opportunities available after spending I is less than $\rho_0 K$, invest in P_0 .

The expected return, $E(R)$, after spending I is:

*Problems similar to this have received considerable attention. J. L. Fisher in Ref. 4 obtains optimal decision rules for the following situations: A decision maker is faced with various investment opportunities over time, and when confronted with a given investment opportunity must accept or reject it--a rejected opportunity is lost forever. The decision maker is assumed to have a fixed budget which can be invested over a given time period. He is not permitted to accumulate investment opportunities and postpone his investment decision. The optimal decision rules specify which investments should be accepted and which should be rejected.

Some work has also been done, Refs. 4 and 5, on the problem of when to stop sampling and accept a specified payoff. In particular the following problems have been solved: Let $\{X_n\}$ be independent random variables with a common distribution function $F(X)$. The X_n are observed sequentially, and if sampling is terminated at X_n , a payoff of either

$$f_n(X_1 \dots X_n) = X_n - Cn \quad (\text{Ref. 4})$$

or

$$f_n(X_1 \dots X_n) = \text{Max } X_n - Cn \quad (\text{Ref. 5})$$

is acquired where $C > 0$ is the cost per observation.

$$\begin{aligned}
 E(R) = & \rho_0(K-I) \int_0^{\rho_0} \int_0^{\infty} \phi_1(\rho_1) \phi_2(k_1) d\rho_1 dk_1 + \\
 & (K-I) \int_{\rho_0}^{\infty} \int_{K-I}^{\infty} \rho_1 \phi_1(\rho_1) \phi_2(k_1) d\rho_1 dk_1 + \\
 & \int_{\rho_0}^{\infty} \int_0^{K-I} [\rho_1 k_1 + (K-I-k_1)\rho_0] \phi_1(\rho_1) \phi_2(k_1) d\rho_1 dk_1
 \end{aligned}$$

If ρ_1 is between zero and ρ_0 , the first integral, then regardless of the value of k_1 , P_0 is preferred to this investment opportunity. However, only $K-I$ funds can be invested in P_0 after information is purchased.

If ρ_1 exceeds ρ_0 and k_1 exceeds $K-I$, the second integral, investing in subsystem 1 is preferred to P_0 and $K-I$ can be invested in subsystem 1 at an expected rate of return, $E(\rho_1 | \rho_1 > \rho_0)$.

If ρ_1 exceeds ρ_0 and k_1 is between zero and $K-I$, the third integral, $E(k_1 | 0 < k_1 < K-I)$ could be invested in subsystem 1 at an expected rate of return, $E(\rho_1 | \rho_1 > \rho_0)$ and the remainder of $K-I$, $K-I-E(k_1 | 0 < k_1 < K-I)$ can be invested in P_0 at ρ_0 .

At each stage of the process a similar expectation can be computed. The expected return of another piece of information after m purchases have been made will depend upon the known values of k_1 and $\rho_1, i=1, \dots, m$, as well as ρ_0 , K and I . Given these values the computation of the expected return is straightforward. This expectation is compared with the

best return that can be made from known investment opportunities. If this expectation exceeds this best return, another piece of information is purchased. If the expected return is less than the best return already available, the sampling process is terminated and the remaining product improvement funds are invested in the best of the known prospects.

This stopping rule is sensible only if the expected gain from an additional purchase of information declines as more information is purchased. If this were not true, then it might pay to take two or more additional observations even though the cost of one more observation exceeded its expected return. A stage by stage decision rule would be invalid. In the process described here, it is true that the marginal value of information is a non-increasing function of the number of observations. This is illustrated in Fig. 1 where the investment opportunity curves are shown for various sample sizes. The investments are ranked in the order of attractiveness. For example, in curve $n=3$, the most favorable investment is $k=k_1$, and $\rho = R_1/k_1$, the next best is $k=k_2$, $\rho = R_2/k_2$ and the least favorable is $k=K-k_1-k_2$ and $\rho = \rho_0 = \frac{R_3}{K-k_1-k_2}$. Assuming that the cost of information is zero, as the sample size increases the investment opportunity curve moves upward, the total return to the K resources increases. For example, the total return that can be obtained from the K funds is R_4 after n_1 observations and R_5 after n_2 observations.* Consider an investment opportunity which is the n_1+1 st observation. Suppose that by investing in this opportunity the n_1 curve evaluated at K is raised from R_4 to R_4^1 . For any investment opportunity curve with $n > n_1$, the value of this n_1+1 st observation can be no greater than its

*For purposes of exposition, these curves are assumed to be continuous.

value to the n_1 portfolio. This follows from the fact that any opportunity available to the n_1 portfolio will be available to the n_2 portfolio. Therefore, the investment which n_1+1 replaces in n_2 portfolio can be no more valuable than the investment which it replaces in the n_1 portfolio. But this is true of every possible investment opportunity which could be revealed by n_1+1 st observation: its value at n_1 must be no less than its value at n_2 . Therefore, the expected return from an additional observation at n_1 must be greater or equal to its expected return at n_2 . A fortiori, this is true if the cost, I , of each observation is greater than zero; i.e., if the cost of an additional observation exceeds its expected return at n_1 , the cost of an additional observation at n_2 will exceed its expected return.

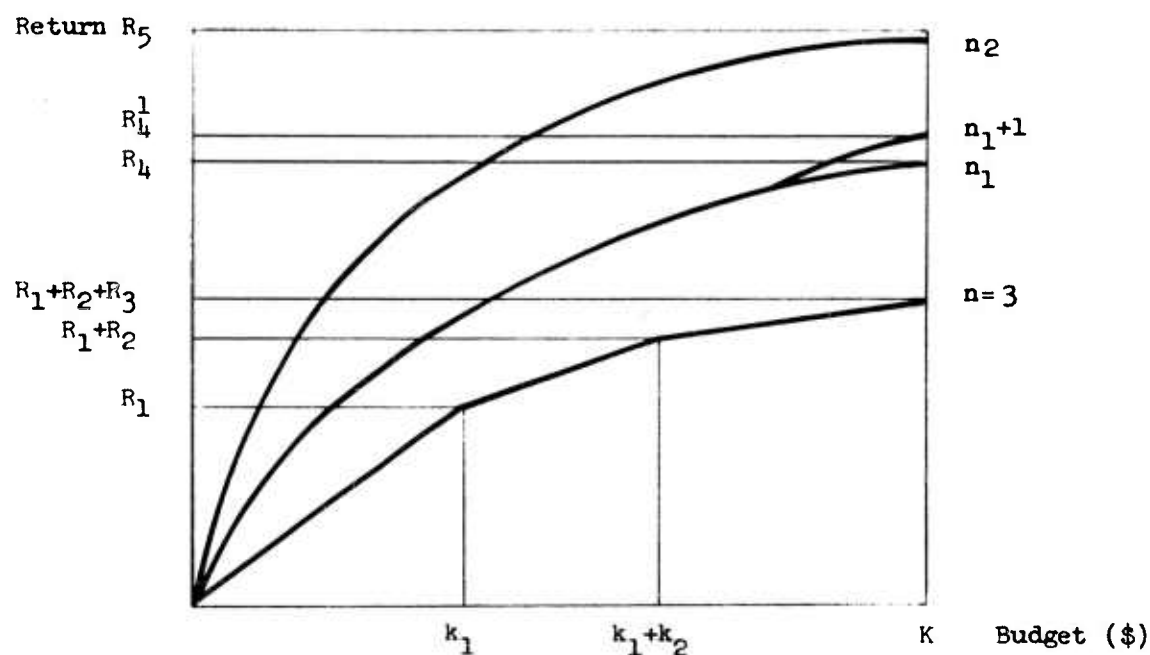


Fig 1

III. A STOPPING RULE WHEN THERE IS SOME PRIOR INFORMATION OF RETURNS

The case is now considered where the n subsystems can be ranked by the amount of mischief each is causing, where mischief is measured as follows. Let λ_i be the exponential failure rate of the i th subsystem. Whenever the i th subsystem fails the weapon system is out of commission for D_i periods and a financial maintenance charge of C_i is incurred. Assuming that the subsystems fail independently, the weapon system failure rate, λ , is the sum of the individual subsystem failure rates,

$$\lambda = \sum_{i=1}^n \lambda_i .$$

The mean time-between-failures, μ , of the weapon system is

$$\mu = \frac{1}{\lambda}$$

The expected weapon system downtime, D , at each failure is

$$D = \mu \sum_{i=1}^n \lambda_i D_i$$

The expected maintenance charge, C , at each failure is

$$C = \mu \sum_{i=1}^n \lambda_i C_i$$

The weapon system amortization rate, α , is

$$\alpha = \frac{K_0}{H} ,$$

where K_0 is the initial cost of the weapon system and H is its expected service life.

It follows that the ratio, M , of expected cost, L , during a cycle to expected good time, T , during a cycle, the cost per unit of weapon system uptime, is

$$M = \frac{L}{T}$$

where

$$L = \alpha(H+D) + C$$

and

$$T = 1/\mu$$

Therefore, M can be written as

$$M = \alpha + \alpha \sum_{i=1}^n \lambda_i D_i + \sum_{i=1}^n \lambda_i C_i$$

The higher M , the less satisfactory is weapon system performance. The contribution, S_i , of the i th subsystem to M is simply

$$S_i = \alpha \lambda_i D_i + \lambda_i C_i$$

or if there are d_i subsystems of type i , then the contribution, S_i^+ , of the i th subsystem is

$$S_i^+ = d_i S_i$$

The value of S_i^+ measures the cost per unit of weapon system good time which the i th subsystem is imposing on the weapon system. It also measures the maximum return obtainable from improvement expenditures on the i th subsystem. This maximum return would occur if λ_i were reduced to zero or if both D_i and C_i were reduced to zero.

As an example, suppose that a weapon system is composed of four subsystems, one of type 1, one type 2, and two of type 3. The failure rates for the three different subsystems are, respectively, $\lambda_1 = 12$ failures per year, $\lambda_2 = 14$ failures per year and $\lambda_3 = 10$ failures per year; the initial cost of the weapon system is, $K_0 = \$1$ million and its expected service life is $H = 5$ years, so that $\alpha = \frac{K_0}{H} = \$200,000$ per year; the down-times incurred because of a failure of subsystem 1, subsystem 2, or subsystem 3 are, respectively, $D_1 = .001$ years, $D_2 = .002$ years, and $D_3 = .004$ years; finally, the maintenance charges resulting from a failure of subsystem 1, subsystem 2, or subsystem 3 are, respectively, $C_1 = \$1000$, $C_2 = \$800$, and $C_3 = \$700$. Under these conditions, the cost per unit of weapon system good time which each of the subsystems (1, 2, and 3) imposes on the weapon system are, respectively,

$$S_1^+ = d_1 (\alpha \lambda_1 D_1 + \lambda_1 C_1) = \$14,400$$

$$S_2^+ = d_2 (\alpha \lambda_2 D_2 + \lambda_2 C_2) = \$16,800$$

and

$$S_3^+ = d_3 (\alpha \lambda_3 D_3 + \lambda_3 C_3) = \$30,000$$

In terms of the previous section, if k_i is distributed as before, then the distribution of ρ_i will be truncated at $\frac{S_i^+}{K_i}$. Given k_i , the maximum rate of return from these k_i funds is the maximum return divided by k_i . Assuming that the distribution of ρ_i is the same as before except for this truncation, then given k_i , the larger S_i^+ , the greater is the expected value of ρ_i -- the mean of a distribution truncated at \int increases with \int . For this reason it is now advantageous to order purchases of

information according to S_i^+ .^{*} That subsystem with the highest S_i^+ should be investigated first; the subsystem with the second highest value of S_i^+ should be investigated next, etc., until the decision maker stops purchasing information.

In terms of the previous example, $S_1^+ = \$14,400$, $S_2^+ = \$16,800$, and $S_3^+ = \$30,000$, which implies that subsystem 3 should be investigated first, then subsystem 2 and finally subsystem 1. As before, the decision maker stops collecting information when the expected return after spending another I on information is less than the return available prior to this purchase.

* The Air Force, realizing that it would be uneconomical to gather information on every subsystem, has developed two methods for isolating product improvement candidates. The first relies solely on the behavior of the subsystem failure rate over time. A subsystem whose failure rate is out of control with respect to some standard failure rate is automatically a candidate for product improvement; i.e., k_i and ρ_i are estimated for out-of-control subsystems. The second method isolates those subsystems which are the highest manhour consumers -- the subsystem whose replacement and repair requires the highest labor expenditure in manhours is the most lucrative candidate for product improvement, etc. Neither method considers downtime, number of similar subsystems in weapon system or the amortization cost. In addition the first neglects maintenance cost while the second neglects failure rate.

IV. CONCLUSION

This Memorandum has considered the relation between information collection and product improvement. In particular, the following problem was considered. Funds of amount K are made available for the improvement of a designated weapon system. The weapon system is composed of n subsystems which fail independently. Each of these subsystems is an investment prospect. If information regarding the attractiveness of each of these prospects were free, cost and return estimates would be obtained for each of the n subsystems and the K funds would be allocated on the basis of these estimates. However, such information is frequently costly. When this is the case, it is usually uneconomical to purchase cost and benefit estimates for each subsystem. The funds expended on excessive information gathering could have been more profitably invested in actual product improvement. This Memorandum has developed a rule whereby the decision maker can determine when to stop gathering information and initiate product improvement.

In the course of developing this rule, a simple technique was suggested for ranking product improvement candidates (subsystems). The ranking of a particular subsystem depended on (1) the subsystem's failure rate, (2) the number of subsystems of this type in the weapon system, (3) the cost in weapon system downtime of a subsystem failure, (4) the maintenance cost of a subsystem failure, (5) the initial cost of the weapon system and (6) the weapon system's expected service life.

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